

Capability Analysis for a Multi-Process Product with Bilateral Specifications

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Process capability indices (PCI) can be viewed as effective and excellent means of measuring product quality and performance. Several capability indices including C_p , C_{pk} , and C_{pm} are well accepted and the implementation of these techniques is successful. The indices C_{pu} , C_{pl} , and C_{pk} have been used previously for evaluating a multi-process product with smaller-the-better, larger-the-better, and nominal-the-best specifications. For a multi-process product with only the nominal-the-best specifications, the evaluation method should be improved, as index C_{pk} cannot reasonably reflect the expected process loss. In this paper, index C_{pmk} is selected to replace C_{pk} and the application is extended to evaluate simultaneously a multi-process product with both symmetric and asymmetric tolerances. An integrated process capability index for a multi-process product with the nominal-the-best specifications is proposed. The relationship between the process capability index and the process yield is introduced. A multi-process capability analysis chart (MPCAC), reasonably reveals the status of the process capability for the entire product, is constructed for practical application. A case study of a Sea island micro-fiber product is provided as of a practical application.

Keywords: Asymmetric tolerances; Multi-process product; Process capability indices

1. Introduction

Process yield, process expected loss and process capability indices (PCIs) are three basic parameters, which have been widely applied for measuring product potential and performance. Of the three, process capability indices are easily understood and can be straightforwardly applied in manufacturing industry. A larger process capability index implies a higher process yield and a lower process expected loss, therefore,

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process capability indices can be viewed as effective and excellent means for measuring product quality and performance. The implementation of these techniques has been successful. Companies have benefited from the use of statistical methods to improve quality and reduce costs. Many engineering designers and shop floor controllers use process capability indices as communication indicators to evaluate and improve the manufacturing processes. For example, process capability indices can assist in solving manufacturing problems when engineering designers negotiate with shop floor supervisors on manufacturing problems, and sales departments and customers can communicate with each other about product characteristics via process capability indices. Customers normally preset product specifications, and a mutually agreed quality level is required to establish understanding and communication between customers and manufacturers through process capability indices.

Process capability indices have been used widely to measure product qualities meeting the required specifications in automotive, semiconductor, and IC assembly manufacturing industries. Statisticians and quality engineers [1–8] studied process capability indices and proposed more effective methods for evaluating process potential and performance.

The first well-known process capability index C_p , introduced by Juran et al. [9], measures the process capabilities in terms of process variation only and does not consider process location, and a value of 1.33 has become the standard benchmark index value for a capable process. C_p is defined as

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{d}{3\sigma}$$

The second process capability index, C_{pk} , was created to offset some of the weakness in C_p . C_{pk} , proposed by Kane [1], quantifies the process capability for the worst half of the data and a value of 1.33 is still considered as the standard minimal boundary for C_{pk} . C_{pk} is defined as

$$C_{pk} = \min \left\{ \frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right\} = \frac{d - |\mu - m|}{3\sigma}$$

where USL and LSL are the upper specification limit and the lower specification limit, respectively, μ is the process mean, σ is the process standard deviation, and $d = (USL - LSL)/2$ is the half interval length and $m = (USL + LSL)/2$ is the midpoint

of the specification interval. For the general form commonly used, “the larger the better” is the rule. According to Boyles [7], C_p and C_{pk} are capability indices with respect to process yield, and are irrelevant to the process target (T). C_p and C_{pk} may fail to account for process centring. Chan et al. [2] proposed an index C_{pm} to take account of process centring. C_{pm} is defined as follows:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{(\sigma^2 + (\mu - T)^2)}} = \frac{d}{3\sqrt{(\sigma^2 + (\mu - T)^2)}}$$

Pearn et al. [5] introduced a single third-generation index C_{pmk} , which is a combination of C_{pk} and C_{pm} . The properties, generalisations, and applications of the index C_{pmk} were introduced by Chen and Pearn [10], Pearn et al. [11], and Pearn et al. [12]. The index C_{pmk} is defined as

$$C_{pmk} = \frac{\min(USL - \mu, \mu - LSL)}{3\sqrt{(\sigma^2 + (\mu - T)^2)}} = \frac{d - |\mu - m|}{3\sqrt{(\sigma^2 + (\mu - T)^2)}}$$

For symmetric tolerances, C_{pmk} provides an accurate measure for process performance, but for asymmetric tolerances, none of the indices mentioned above can provide consistent and reasonable measures of process capabilities. The generalisation of C_{pmk} was proposed by Pearn et al. [11] to reflect process capability with asymmetric tolerances. For general application, to assess a multi-process product, we revise the definition of the index as

$$C''_{pmk} = \frac{d^* - A}{3\sqrt{(\sigma^2 + A^2)}}$$

where $A = \max \{d^*(\mu - T)/D_u, d^*(T - \mu)/D_l\}$, $D_u = USL - T$, $D_l = T - LSL$, and $d^* = \min \{D_u, D_l\}$. $A = |\mu - T|$ when $T = m$ (symmetric case), and C''_{pmk} degenerates to the original index C_{pmk} . The factor A ensures that the new generalisation C''_{pmk} obtains its maximal value at $\mu = T$ (process is on-target) regardless of whether the tolerances are symmetric ($T = m$) or asymmetric ($T \neq m$). For a fixed σ the C''_{pmk} value decreases when μ shifts away from T . In reality, the C''_{pmk} value decreases faster when μ moves away from T to the closer specification limit, and decreases more slowly when μ moves away from T to the farther specification limit. Chen and Pearn [10] proposed that when the value of C''_{pmk} is given, the lower bound of the process yield can be calculated using the formula $p \geq 2\Phi(3C''_{pmk}) - 1$. As an example, the process yield is at least greater than 99.73% when C''_{pmk} equals 1. The statistical properties such as estimation, distributions, and moments about the process capability index C''_{pmk} have been established by Pearn et al. [11].

Chen et al. [13] has applied the indices C_{pu} , C_{pl} , and C_{pk} for evaluating the process capability for a multi-process product with smaller-the-better, larger-the-better, and nominal-the-best specifications. For a multi-process product with nominal-the-best specifications only, the evaluation method can be improved. In particular, index C_{pk} is irrelevant to the process target (T) and may fail to account for process centring. C_{pk} cannot reasonably reflect the process expected loss. In this paper, an index C_{pmk} is selected to replace C_{pk} . Since most products are designed with many symmetric and asymmetric

Table 1. The key specifications of Sea island micro-fiber TS2-135N.

Product characteristic	Specification
1. Denier	3.5 ± 0.2
2. Tenacity	5.0 ± 0.5
3. Elongation	50 ± 5%
4. OPU	1.5 ± 0.5%
5. Crumple number	11 - 1, 11 + 2
6. Crimple	12 ± 2%
7. Rate of crimple elasticity	45 ± 10%
8. Water	3 - 0.5%, 3 + 0.3%
9. Hot-air shrinkage	5 - 2%, 5 + 1%
10. Length	51 ± 6%

bilateral tolerances, and based on the advantages of C''_{pmk} , this index will be used in the paper. A multi-process capability analysis chart (MPCAC), for revealing the status of the process capability for an entire product with nominal-the-best specifications, is constructed for practical application. The methodology proposed by Sung et al. [14] was used for evaluating the multi-process capability. The approach in this paper is to propose a concise and applicable methodology for measuring the integrated process capability for an entire product with many symmetric and asymmetric tolerances.

2. Product Description – Sea Island Micro-Fiber

High-quality micro-fiber artificial leather can be used for many products, such as sofas, casual shoes, sports goods, gloves, linings, handbags, and clothes. The second business department of the Yanin Leather Co. Ltd, located in central Taiwan, produces PU synthetic leather products. In order to raise Yanin’s competitive position, changing the PU product structure to a high quality non-woven backing substrate PU artificial leather is crucial. They invested in two lines of European made non-woven-producing equipment in 1996 and 1999 at the Lukang plant, and invested in Japanese-made Sea-island micro-fiber spinning equipment in 2000 to produce high-value-added products. Yanin owns a complete set of micro-fiber artificial leather-producing equipment. The products have the unique structure of genuine leather and soft feel, they are lightweight, have a silk feel, brilliant colour range, and various embosses. Those characteristics make them suitable for a wide range of uses. Table 1 gives the specifications of the key characteristics of Sea island micro-fiber and the production process of Sea island micro-fiber is described in Fig. 1.

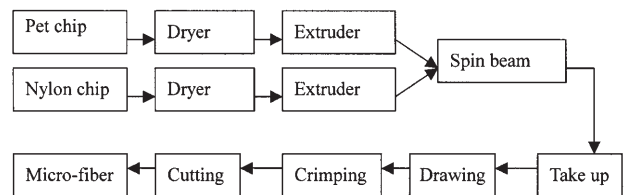


Fig. 1. The production process of Sea island micro-fiber.

Sea island micro-fiber is packed in 230 g bags, and 5 g is taken from every bag for conventional inspection to examine whether all the characteristic meet the preset quality specifications. If any characteristic does not meet the preset specification, further quality investigation or improvement action will be taken to eliminate the unacceptable process capability.

3. Process Capability Index and Process Yield for a Multi-Process Product

According to Boyles [7], there is a one-to-one mathematical relationship between the index S_{pk} and the process yield. When $S_{pk} = c$, process yield $\%yield = 2\Phi(3c) - 1$. The index S_{pk} is defined as

$$S_{pk} = \frac{1}{3} \Phi^{-1} \left\{ \frac{1}{2} \Phi \left(\frac{USL - \mu}{\sigma} \right) + \frac{1}{2} \Phi \left(\frac{\mu - LSL}{\sigma} \right) \right\}$$

where Φ denotes the standard normal cumulative distribution function.

Since $C''_{pmk} \leq S_{pk}$, $C''_{pmk} = c$ guarantees $S_{pk} \geq c$ and implies that $P_j \geq 2\Phi(3c) - 1$. Although the Sea island micro-fiber product is illustrated in this paper, for general application, a product with w quality characteristics is introduced in this section. The process yield of the end product consists of the number of w quality characteristics and can be expressed as $P^T \geq 2\Phi(3C''_{pmk}) - 1$, $j = 1, 2, \dots, w$. We intend to define an integrated process capability index C^T_k to express the integrated process capability of the entire product with w quality characteristics.

$$C^T_k = \frac{1}{3} \Phi^{-1} \left(\left(\left\{ \prod_{j=1}^w 2\Phi(3C''_{pmk}) - 1 \right\} + 1 \right) / 2 \right)$$

When $C^T_k = v$, solving the above equation we have

$$\prod_{j=1}^w 2\Phi(3C''_{pmk}) - 1 = 2\Phi(3v) - 1$$

Customers usually predetermine the levels of the product quality, and they will be satisfied when all the product quality characteristics exceed or meet their expectations. If some of the product quality characteristics are below their expectations, they might return the product or ask for a replacement. Where the integrated process capability for a multi-process product is concerned, the integrated process capability is lower than any individual process capability. Similarly, the process yield of the end product with a multi-process is lower than any individual process yield ($P^T \leq P_j$). When the entire process yield (or entire product capability) is preset to satisfy the required level, the individual process yield (or individual process capability) should exceed the preset standard for the entire product. Assume the quality levels for each characteristic are independent, the process yield (P^T) of the end product becomes

$$P^T = \prod_{j=1}^w P_j \geq \prod_{j=1}^w 2\Phi(3C''_{pmk}) - 1 = 2\Phi(3v) - 1$$

There exists a one-to-one mathematical relationship between the integrated process capability index of the entire product

with several quality characteristics and the process yield of the end product. A greater integrated process capability index of the entire product (C^T_k) corresponds to a higher process yield of the end product (P^T). For instance, when the integrated process capability index $C^T_k = 1.0$ and 1.33, the corresponding process yield of the end product (P^T) equals 99.73% and 99.99%, respectively.

For a general situation, when the integrated process capability index (C^T_k) equals a preset value, say a , we have

$$C^T_k = \frac{1}{3} \Phi^{-1} \left(\left(\left\{ \prod_{j=1}^w 2\Phi(3C''_{pmk}) - 1 \right\} + 1 \right) / 2 \right) = a$$

Customers expect good quality levels for all product characteristics. Specifically, when the values of the process capabilities for individual characteristics are all equal to a_0 (the minimal value of the process capability for each quality characteristic), the above formula will become

$$\frac{1}{3} \Phi^{-1} \left(\left(\left\{ \prod_{j=1}^w 2\Phi(3a_0) - 1 \right\} + 1 \right) / 2 \right) = a$$

Hence, the value a_0 for the individual process capability can be attained by solving the previous equality when the integrated process capability equals a , where

$$a_0 = \frac{1}{3} \Phi^{-1} \left(\frac{w\sqrt{2\Phi(3a) - 1} + 1}{2} \right)$$

When the value of the individual process capability index (a_0) is determined, the corresponding process yield for each quality characteristic is found and the end process yield (P^T) can be calculated. The value of the individual process capability index (a_0) must be greater than the value of the integrated process capability index for the entire product, to guarantee that the end process yield will meet the pre-required standard. For a product with four quality characteristics, if the process capability indices for four quality characteristics equal 1.133, the integrated process capability index for the entire product will be 1.0 and the corresponding process yield of the end product is 99.73%. If the process capability indices for four quality characteristics equals to 1.00, the corresponding process yield of the individual quality characteristic is 99.73%, but the process yield of the end product is only approximately 98.92%. The value of the individual process capability index (a_0) can be obtained through statistical software such as SAS, SPSS, and Statistic. The SAS program for calculating the individual process capability value is provided in the Appendix.

4. The Process Capability Analysis Chart for a Multi-Process Product

In the preceding section, we mentioned the process capability index S_{pk} , proposed by Boyles [7]. Vännman [15] introduced a process capability plot to define the capability of the process, called the (δ, γ) -plot, where $\delta = (\mu - T)/d$, and $\gamma = \sigma/d$. The (δ, γ) -plot is an effective graphical method for theoretically comparing and contrasting different process capability indices and is invariable with respect to the value of the specification

limits. Hence, the revised (δ, γ) -plot can be applied to compare the process capabilities for the multiple quality characteristics of a product. Boyles [7] proposed a contour map to evaluate the process capabilities for a multi-process product. The status of the process capability including the process precision and the process accuracy is evaluated in terms of location on the contour map. The contour map is not applicable when the specifications of the multi-process product are different. Products with multi-process characteristics are usually designed with different specifications. To avoid the limitation and based on Boyles's contour map [7], we have revised the (δ, γ) -plot to evaluate the process capabilities for a multi-process product with different specifications.

The approach is to replace μ and σ on the X -axis and Y -axis of the (δ, γ) -plot by $X_a = (\mu - T)/D_u$ if $\mu \geq T$ or $X_a = (\mu - T)/D_l$ if $\mu \leq T$ and $Y_p = \sigma/d^*$, respectively. From the transformation formula of X_a , X_a becomes $-1, 0$, and 1 when $\mu = LSL$, $\mu = T$, and $\mu = USL$, respectively. The reason for doing that is to standardise the different specifications of many quality characteristics to transform the X_a value to within $(-1, 1)$ to compare effectively and efficiently the multi-process capabilities in combination. According to the locations of values of (X_a, Y_p) on the X - Y dimension, which represents the process capabilities of multiple characteristics, the multi-process capabilities of a product can be assessed simultaneously. We call the new process capability analysis chart the multi-process capability analysis chart (MPCAC).

The relationship between C''_{pmk} and (X_a, Y_p) is as follows

$$C''_{pmk} = \frac{1 - |X_a|}{3\sqrt{Y_p^2 + X_a^2}}$$

Based on the predetermined process capability index for an individual quality characteristic a_0 , mentioned in the preceding section, bold contour lines, for example $a_0 = 1.0, 1.33$, can be added on the MPCAC to differentiate quality levels. In addition, seven vertical lines, representing X_a values from $-1, -0.5, -0.25, 0, 0.25, 0.5, 1.0$, labelled L3, L2, L1, T, U1, U2, and U3 from left to right (Fig. 2) on the dimensional space reflect the degree of shift for process targeting. Motorola's "six-

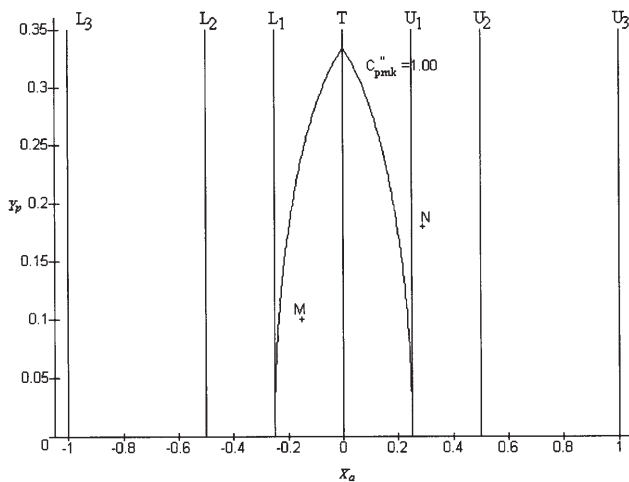


Fig. 2. A multi-process capability analysis chart (MPCAC).

sigma" program requires that when the process mean is in control, it will not be closer than six standard deviations from the nearest specification limit. Six sigma is translated to 3.4 defects per million opportunities. A process which allows process mean shifts of 1.5 standard deviations satisfies Motorola's requirement if $C_{pk} \geq 1.5$ and $C_p \geq 2.0$. The corresponding degrees of shift for the seven vertical lines from left to right are $-6\sigma, -3\sigma, -1.5\sigma$, on target, and $1.5\sigma, 3\sigma$, and 6σ according to the allowable shift from Motorola's standard. For example, assume the process capability of quality characteristics M and N are known and the value of the process capability is marked on Fig. 1. It is easy to tell that process M has a better process capability than process N . The targeted degrees and process deviations are widely different. Process M has a better targeting status and smaller process variation than process N .

The strengths of the MPCAC are summarised as follows:

1. Removing the limitation of the other methods, MPCAC simultaneously assesses many quality levels on the same chart for a multi-process product.
2. It is easy to differentiate the quality levels of a multi-process product with respect to the locations of the process capability values on the MPCAC.
3. MPCAC reasonably reflects the degree of process targeting for a multi-process product.
4. MPCAC help engineers to decide on quality improvement requirements e.g. whether to elevate the process precision, or to improve the process accuracy, or to improve both of them, according to the locations of the process capability values on the MPCAC.
5. MPCAC can effectively and efficiently evaluate the integrated process capability of the entire multi-process product.

5. An Application

The following case is introduced to illustrate the application of MPCAC. The Yanin Leather Co, produces PU synthetic leather products. The Sea island micro-fiber artificial leather can be applied to many products, such as sofas, casual shoes, sports goods, gloves, linings, handbags, and clothes. The specifications of the 10 key characteristics of Sea island micro-fiber are nominal-the-best with 7 symmetric and 3 asymmetric tolerances, which were mentioned in Section 2. The original inspection procedure is time consuming and costly and quality investigations and improvement plans for non-conforming products have exhausted the engineers. We used the new methodology to perform quality evaluation for 10 characteristics simultaneously and effectively and to efficiently search out the unacceptable items.

From the preceding sections, first, a value for the integrated process capability (C'_i) is determined. The minimal acceptable value is above 1.00 to guarantee that the process yield for the end product is greater than 99.73%. Secondly, the corresponding individual process capability for each quality characteristic (a_0) is 1.214, which can be obtained using the formula $a_0 = \frac{1}{3} \Phi^{-1} ((10\sqrt{2\Phi(3) - 1}) + 1)/2$ or the SAS program in the Appendix. The inspection data for the 10 quality characteristics

Table 2. Process capabilities for Sea-island micro-fiber TS2-135N.

Product characteristic	LSL	T	USL	μ	σ	X_a	Y_p	C''_{pmk}
1. Denier	3.3000	3.5000	3.7000	3.4900	0.0500	-0.0500	0.2500	1.2421
2. Tenacity	4.5000	5.0000	5.5000	4.8000	0.1500	-0.4000	0.3000	0.4000
3. Elongation	47.5000	50.0000	52.5000	50.7500	0.2035	0.3000	0.0814	0.7506
4. OPU	1.4925	1.5000	1.5075	1.4992	0.0015	-0.1067	0.2000	1.3137
5. Crumple number	10.0000	11.0000	12.0000	11.1245	0.0341	0.1245	0.0341	2.2608
6. Crimple	11.7600	12.0000	12.2400	12.0150	0.0500	0.0625	0.2083	1.4367
7. Rate of crimple elasticity	40.5000	45.0000	49.5000	46.5500	0.3575	0.3444	0.0794	0.6182
8. Water	2.9850	3.0000	3.0090	2.9980	0.0002	-0.1333	0.0222	2.1372
9. Hot-air shrinkage	4.9000	5.0000	5.0500	4.9800	0.0090	-0.2000	0.1800	0.9911
10. Length	47.9400	51.0000	54.0600	50.6250	0.3500	-0.1225	0.1144	1.7448

is given in Table 2. Figure 3 shows the quality status for the 10 key characteristics of the Sea island micro-fiber TS2-135N.

According to the above inspection data, the integrated process capability (C''_k) of the Sea island micro-fiber product is not acceptable because 4 process capability indices of the quality characteristics are not within the contour line. Quality improvement action must be taken immediately to enhance product quality. Based on Table 2 and Fig. 3, the unacceptable quality process capabilities for Sea island micro-fiber products are discussed below:

1. Tenacity (a2): the process variation of tenacity is too high and the process shift is too far away from the target. Quality improvement action including both diminishing the process variation and shifting the process centre to become on target must be taken immediately to improve the poor process capability of tenacity.
2. Elongation (a3) and rate of crimple elasticity (a7): the C''_{pmk} values for both processes are moderately below expectation. Although the process variations of elongation and rate of crimple elasticity are acceptable, the processes not centred on the process targets make the C''_{pmk} values below expectation. All that needs to be done for elongation and rate of crimple elasticity is to shift the process centres to the target to improve the process capabilities. The index C_{pk} is irrelevant to the process target (T) and may fail to account

for process centring. To reveal the strength of the index C''_{pmk} , the C_{pk} values 2.8665 and 2.7506 for elongation (a3) and rate of crimple elasticity (a7), respectively, are also calculated. The two process capabilities are superior when index C_{pk} is used. From Motorola's requirement, the process shifts are more than 1.5σ for elongation (a3) and rate of crimple elasticity (a7).

3. Hot-air shrinkage (a9): the C''_{pmk} value is unacceptable. To enhance the quality level, either improving the process targeting or trying to reduce the process variation is required.

Once all the above process capabilities of the unacceptable process characteristics are improved, the integrated process capability of the entire Sea island micro-fiber can meet the pre-required quality level ($C''_k = 1.0$), and the process yield of the end product can be guaranteed ($P^T \geq 99.73\%$).

6. Conclusions

Process capability indices are easily understood and can be straightforwardly applied in the manufacturing industry. In this paper, we extend this application to evaluate simultaneously a multi-process product with both symmetric and asymmetric tolerances. An integrated process capability index for a multi-process product is proposed. A multi-process capability analysis chart (MPCAC) is constructed for practical application. It shows the status of process capability for an entire product with nominal-the-best specifications. The relationship between process capability index and the process yield is also introduced. A Sea island micro-fiber product with 7 symmetric and 3 asymmetric quality characteristics is introduced as a case study. Based on the methodology we proposed, the integrated product capability is justified and the unacceptable quality characteristics are effectively and efficiently disclosed. Quality improvement plans can be easily initiated to enhance the entire product quality.

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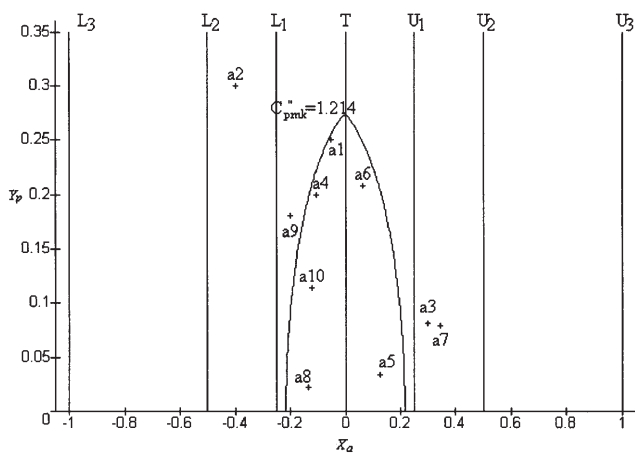


Fig. 3. The quality status for Sea island micro-fiber TS2-135N (a1 denotes the C''_{pmk} value of quality characteristic 1).

Appendix

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/*****;/
/*          SAS PROGRAMING          */;
/*          The calculation of individual process */;
/*          capability for a multi-process product */;
/*****;/
OPTIONS REPLACE PAGESIZE=78 LINESIZE=78 NODATE;
DATA A0VALUE;
CTK=1.000; /*The integrated process capability index for the multi-process
product*/;
N=15; /*The number of quality characteristics of the multi-process
product*/;
DO R=1 TO N BY 1;
I=1/R; F=(1+(2*PROBNORM(3*CTK)-1)**I)/2; A0=PROBIT(F)/3;
OUTPUT;
END;

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